

Solutions to Mathematics Workbook-1 | Quadratic Equations & Inequalities [2022]

Level - 2	Daily Tutorial Sheet 6
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76.(C) α, β are the roots of $ax^2 + 2bx + c = 0 \Rightarrow \alpha + \beta = -\frac{2b}{a}, \alpha\beta = \frac{c}{a}$

$\alpha + \delta, \beta + \delta$ are the roots of $Ax^2 + 2Bx + C = 0 \Rightarrow \alpha + \delta + \beta + \delta = -\frac{2B}{A}, (\alpha + \delta)(\beta + \delta) = \frac{C}{A}$

Now $(\alpha - \beta)^2 = [(\alpha + \delta) - (\beta + \delta)]^2 \Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = (\alpha + \delta + \beta + \delta)^2 - 4(\alpha + \delta)(\beta + \delta)$

$\Rightarrow \frac{4b^2}{a^2} - \frac{4c}{a} = \frac{4B^2}{A^2} - \frac{4C}{A} \Rightarrow \frac{b^2 - ac}{a^2} = \frac{B^2 - AC}{A^2} \Rightarrow \frac{b^2 - ac}{B^2 - AC} = \frac{a^2}{A^2}$

77.(A) $k(6x^2 + 3) + rx + 2x^2 - 1 = 0 \Rightarrow (6k + 2)x^2 + rx + 3k - 1 = 0$

$6k(2x^2 + 1) + px + 4x^2 - 2 = 0 \Rightarrow (12k + 4)x^2 + px + 6k - 2 = 0$

As these equations have both roots common $\frac{6k+2}{12k+4} = \frac{r}{p} = \frac{3k-1}{6k-2} \Rightarrow \frac{r}{p} = \frac{1}{2}$ or $2r - p = 0$

78.(B) Let α be the common root

$\Rightarrow 3\alpha^2 + a\alpha + 1 = 0$ and $2\alpha^2 + b\alpha + 1 = 0$

Multiplying the first equation by 2, the second equation by 3 and then subtracting the second equation from the first equation we get : $\alpha = \frac{1}{2a - 3b}$

Putting this value of α in $2\alpha^2 + b\alpha + 1 = 0$ we get $5ab - 2a^2 - 3b^2 = 1$

79.(A) For the equation $2a^2x^2 - 2abx + b^2 = 0, D = (-2ab)^2 - 4(2a^2)(b^2)$

$= -4a^2b^2 < 0 \Rightarrow$ The equation has both complex roots.

For the equation $p^2x^2 + 2pqx + q^2 = 0, D = (2pq)^2 - 4p^2q^2 = 0$

$\Rightarrow$ The equation has both real roots.

Since the first equation has both roots complex and the second equation has both roots real, there is no common root between these equations.

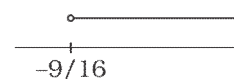
80.(C) $y = \frac{ax^2 + 3x - 4}{a + 3x - 4x^2} \Rightarrow x^2(a + 4y) + (3 - 3y)x - 4 - ay = 0$

As $x \in R, D \geq 0$ Now, $D = (3 - 3y)^2 - 4(a + 4y)(-4 - ay) \geq 0 \Rightarrow (16a + 9)y^2 + (46 + 4a^2)y + (16a + 9) \geq 0$

'y' can take values which satisfy this inequality. Thus, 'y' will take all real values if this inequality is true for all values of 'y'.

A quadratic in y is positive if coeff. of $y^2 > 0$ and $D \leq 0$.

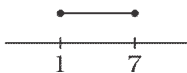
$\Rightarrow 16a + 9 > 0 \Rightarrow a > -\frac{9}{16} \dots \dots \text{(i)}$

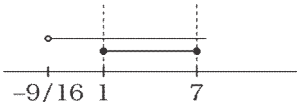


and $D \leq 0 \Rightarrow (46 + 4a^2)^2 - 4(16a + 9)(16a + 9) \leq 0$

$\Rightarrow (23 + 2a^2)^2 - (16a + 9)^2 \leq 0 \Rightarrow (a^2 + 8a + 16)(a^2 - 8a + 7) \leq 0$

$$\Rightarrow (a+4)^2 (a^2 - 8a + 7) \leq 0 \quad \Rightarrow \quad a^2 - 8a + 7 \leq 0 \quad \left[\because (a+4)^2 \geq 0 \forall a \right]$$

$$\Rightarrow (a-1)(a-7) \leq 0$$


Combined solution:  $\Rightarrow a \in [1, 7]$

81.(B) $y = \frac{x+a}{x^2+bx+c^2} \Rightarrow yx^2 + (by-1)x + c^2y - a = 0$

As $x \in R, D \geq 0$

$$\Rightarrow (by-1)^2 - 4y(c^2y-a) \geq 0 \Rightarrow (b^2-4c^2)y^2 + (4a-2b)y + 1 \geq 0 \quad \forall y \in R \text{ because } y \text{ attains all real values}$$

$$\Rightarrow b^2 - 4c^2 > 0 \text{ and } D = (4a-2b)^2 - 4(b^2-4c^2) = 4[(2a-b)^2 - b^2 + 4c^2] \leq 0$$

$$\Rightarrow D = 16[a^2 + c^2 - ab] \leq 0$$

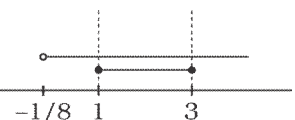
82.(D) $y = \frac{ax^2+x-2}{a+x-2x^2} \Rightarrow (a+2y)x^2 + (1-y)x - 2 - ay = 0$

$$\text{As } x \in R, D \geq 0 \Rightarrow (y-1)^2 - 4(a+2y)(-2-ay) \geq 0 \Rightarrow (8a+1)y^2 + (4a^2+14)y + (8a+1) \geq 0$$

Now, y will take all real values if this inequality is true for all values of y .

$$\Rightarrow 8a+1 > 0 \Rightarrow a > -\frac{1}{8} \text{ and } D \leq 0 \Rightarrow (4a^2+14)^2 - 4(8a+1)^2 \leq 0$$

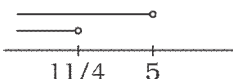
$$\Rightarrow (a^2+4a+4)(a^2-4a+3) \leq 0 \Rightarrow a^2-4a+3 \leq 0 \quad [\because a^2+4a+4 \geq 0] \Rightarrow (a-3)(a-1) \leq 0$$

Combined solution:  $\Rightarrow a \in [1, 3]$

83.(A) $y = \frac{mx^2+3x+4}{x^2+3x+4} < 5 \Rightarrow \frac{mx^2+3x+4-5x^2-15x-20}{x^2+3x+4} < 0 \Rightarrow (m-5)x^2 - 12x - 16 < 0 \quad \forall x \in R$

$$[\because x^2+3x+4 > 0 \text{ as } a=1 > 0 \text{ and } D=3^2-4(4) < 0]$$

$$\Rightarrow a < 0 \Rightarrow (m-5) < 0 \Rightarrow m < 5 \text{ and } D < 0 \Rightarrow 12^2 - 4(-16)(m-5) < 0 \Rightarrow m < \frac{11}{4}$$

Combined solution:  $\Rightarrow m < \frac{11}{4}$

84.(CD) Let $y = \frac{(x-a)(x-c)}{x-b} \Rightarrow x^2 - (a+c+y)x + (ac+by) = 0$ where a, b, c are distinct

By analysing the behaviour of this function, we can predict the behaviour of functions in options (A) and (C)

As $x \in R, D \geq 0$

$$\Rightarrow (a+c+y)^2 - 4(ac+by) \geq 0 \Rightarrow y^2 + 2(a+c-2b)y + (a-c)^2 \geq 0$$

$$\text{Now, coefficient of } y^2 = 1 > 0 \text{ and } D = 4(a+c-2b)^2 - 4(1)(a-c)^2 = 4[(2a-2b)(2c-2b)]$$

$$D = -16(a-b)(b-c)$$

If $a > b > c \Rightarrow D < 0$; Also, If $a < b < c \Rightarrow D < 0$

$\Rightarrow$ 'y' can take all real values.

Moreover, when $(a > b \text{ and } c > b)$ or $(b > a) \text{ and } (b > c)$,

We get $D > 0$.

In that case, $y^2 + 2(a + (-2b))y + (a - c)^2 < 0$ for some values of y and thus y can't take all real values.

Also: $y \in (-\infty, \infty) \Leftrightarrow \frac{1}{y} \in (-\infty, \infty)$ and thus now we can also predict the behaviour of functions in options (B) and

(D)

From the above arguments, we can conclude that only (C) and (D) have range $\mathbb{R}$, while (A) and (B) can't take all real values.

85.(B) $x^2 + ax + b = 0$ [a, b are the roots (given)]

Product of roots $= ab = b \Rightarrow a = 1$ and sum of roots $= a + b = -a \Rightarrow b = -2$ [$\because a = 1$]

Hence, required quadratic equation: $x^2 + x - 2 = 0$

Least value $= \frac{-D}{4A}$ [For $Ax^2 + Bx + C = 0$] $= -\frac{9}{4}$